

Notes on the Private α -core and Coarse α -core in Strategic Form Games

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Abstract

The purpose of this note is to take an example in which the set of coarse α -core strategies does not contain the set of private α -core strategies in strategic form games with differential information. Moreover this remark holds independent of decision stages.

key words: private α -core, α -core, differential information

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1 Introduction

Wilson (1978) proposed how agents within a coalition share their information in pure exchange economies with differential information. In this literature, two extreme situations are mainly considered according to Wilson. First, the members in a coalition pool their information. Second, they can use only their common information, which is called coarse core. Another concept — the private core — is defined by Yannelis (1991). In this private core, players can coordinate their strategies, however, cannot exchange their information.

Yannelis (1991) and Lefevre (2001) proved the existence of the private core in the exchange economies with differential information. Moreover Yannelis (1991) also proved the nonemptiness of private α -core in strategic form games with differential information, which may be regarded as the first formal existence theorem in this literature. In his paper, Yannelis (1991) suggests that “We can show that the set of coarse α -core strategies contains the set of α -core (private α -core) strategies for the game with differential information (p. 190).” However, we can find a counter example to this problem.

The purpose of this paper is to take an example in which the set of coarse α -core strategies does not contain the set of private α -core strategies in strategic form games with differential information. From our example, the existence of private α -core does not necessary imply the existence of coarse

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α -core. The existence of private α -core is not always useful for the existence for the existence of coarse α -core.

The paper is organized as follows. Section 2 presents definitions. In section 3 we take a counter example. We conclude in section 4.

2 Definitions

Let $N = \{1, \dots, n\}$ be the set of players. We denote by \mathcal{N} the set of all nonempty subsets of N , and it is called coalitions. Let Ω be a finite set with the generic element ω . The set Ω represents the states of the world, and the generic element ω is called a state.

Definition 1. A strategic form game with differential information is defined as $\{X_i, u_i, \mathcal{P}_i, \mu_i\}_{i \in N}$, where

1. X_i is the set of strategies for player i ,
2. $u_i : \Omega \times \prod_{i \in N} X_i \rightarrow \mathbb{R}$ is player i 's payoff function,
3. \mathcal{P}_i is a partition of Ω , and
4. μ_i is a strictly positive probability measure on Ω that represents player i 's prior.

For each partition Π of Ω , we denote by $\Pi(\omega)$ the element of Π which contains ω . For each $\sigma \in \prod_{i \in N} X_i^\Omega$, we denote by $Eu_i(\sigma|\mathcal{P}_i)(\omega)$ the conditional expected utility function for player i , which is defined as

$$Eu_i(\sigma|\mathcal{P}_i)(\omega) := \sum_{t \in \mathcal{P}_i(\omega)} \frac{\mu_i(\{t\})}{\mu_i(\mathcal{P}_i(\omega))} u_i(t, \sigma_1(t), \dots, \sigma_n(t)) .$$

As usual, we define the expected utility function for player i as

$$Eu_i(\sigma) := \sum_{\omega \in \Omega} \mu_i(\{\omega\}) u_i(\omega, \sigma_1(\omega), \dots, \sigma_n(\omega)) .$$

An information structure for $S \in \mathcal{N}$ is a collection $(\mathcal{P}_i)_{i \in S}$ for partition of Ω . The meet of partitions $(\mathcal{P}_i)_{i \in S}$ is the finest partition of Ω that is coarser than each \mathcal{P}_i for all i in S , which is denoted by $\bigwedge_{i \in S} \mathcal{P}_i$. This situation describes that each player in S does not exchange their

information at all when they communicate. That is to say, no information is pooled in the coalition S .

For notational convenience, we denote X_i^Ω by Σ_i . As usual, we define $\Sigma_S = \prod_{i \in S} \Sigma_i$ as the set of joint strategies in the coalition S . This representative element is σ_S . To simplify notations we define

$$\Sigma_S^p := \{\sigma_S \in \Sigma_S \mid \sigma_i \text{ is } \mathcal{P}_i\text{-measurable for all } i \in S\}$$

for all $S \in \mathcal{N}$. Moreover we define for all $S \in \mathcal{N}$,

$$\Sigma_S^c := \{\sigma_S \in \Sigma_S \mid \wedge_{i \in S} \mathcal{P}_i\text{-measurable}\}.$$

Definition 2. If there do not exist $S \in \mathcal{N}$ and $\sigma_S \in \Sigma_S^k$ such that for all $\sigma_{N \setminus S} \in \Sigma_{N \setminus S}^k$ and $\omega \in \Omega$, $Eu_i(\sigma_S, \sigma_{N \setminus S} \mid \mathcal{P}_i)(\omega) > Eu_i(\sigma^* \mid \mathcal{P}_i)(\omega)$ for all $i \in S$, then $\sigma^* \in \prod_{i \in \mathcal{N}} \Sigma_i^k$ is called the *private α -core strategy* in the case of $k = p$, the *coarse α -core strategy* in the case of $k = c$, and the *non-measurable coarse α -core strategy* in the case of $\Sigma_S^k = \Sigma_S$ and $\Sigma_{N \setminus S}^k = \Sigma_{N \setminus S}$.

Definition 3. If there do not exist $S \in \mathcal{N}$ and $\sigma_S \in \Sigma_S^k$ such that $Eu_i(\sigma_S, \sigma_{-S}) > Eu_i(\sigma^*)$ for all $\sigma_{N \setminus S} \in \Sigma_{N \setminus S}^k$ and $i \in S$, then $\sigma^* \in \prod_{i \in \mathcal{N}} \Sigma_i^k$ is called the *ex ante private α -core strategy* in the case of $k = p$, the *ex ante coarse α -core strategy* in the case of $k = c$, and the *ex ante non-measurable coarse α -core strategy* in the case of $\Sigma_S^k = \Sigma_S$ and $\Sigma_{N \setminus S}^k = \Sigma_{N \setminus S}$.

3 An Example

The set of players is $\{1, 2\}$. The state set is given by $\{\omega_1, \omega_2\}$. The information structure for player 1 is $\mathcal{P}_1 = \{\{\omega_1\}, \{\omega_2\}\}$. Let $\mathcal{P}_2 = \{\{\omega_1, \omega_2\}\}$. Player 2 places the equal probability on the two states. The payoff matrices are given by the following tables.

1 \ 2	A	B
A	a, a	$0, 0$
B	$0, 0$	b, b

state ω_1

1 \ 2	A	B
A	$0, 0$	b, b
B	a, a	c, c

state ω_2

The left matrix is occurred at state ω_1 , and the right is occurred at state ω_2 . We assume $a > b > 0$. In the strategy bundle (AB, BA) , the first component A is player 1's action at state ω_1 , the second component B is player 1's action at state ω_2 . In the similar way, the third component B is player 2's action at ω_1 . In this example $\Sigma_{\{1\}}^p = \{AA, AB, BA, BB\}$ and $\Sigma_{\{2\}}^p = \{AA, BB\}$. Moreover note that $\Sigma_{\{1,2\}}^c = \{(AA, AA), (AA, BB), (BB, AA), (BB, BB)\}$.

strategy bundle	$Eu_1(\cdot \mathcal{P}_1)(\omega_1)$	$Eu_1(\cdot \mathcal{P}_1)(\omega_2)$	$\frac{1}{2}u_i(\omega_1, \cdot) + \frac{1}{2}u_i(\omega_2, \cdot)$
(AA, AA)	a	0	$a/2$
(AA, BB)	0	b	$b/2$
(BB, AA)	0	a	$a/2$
(BB, BB)	b	c	$b/2 + c/2$
(AB, AA)	a	a	a
(BA, AA)	0	0	0
(AB, BB)	0	c	$c/2$
(BA, BB)	b	b	b

Table 1: expected utility value

Case 1. $c = 0$

Using this table, the strategy bundle (AA, AA) cannot be improved upon by any strategies in $\Sigma_{\{1,2\}}^c$. Then (AA, AA) is the coarse α -core strategy. In the same way, (BB, AA) is also the coarse α -core strategies. (BB, BB) is improved upon by (AA, AA) .

The strategy bundle (AB, AA) cannot be improved upon by any strategies in $\Sigma_{\{1,2\}}^p$. (AA, AA) , (AA, BB) and (BB, BB) are improved upon by (AB, AA) . Then there strategy bundles are not private α -core strategies.

Remark 1

1. The set of private α -core strategies is $\{(AB, AA)\}$.
2. The set of coarse α -core strategies is $\{(AA, AA), (BB, AA)\}$.

Remark 2

1. The set of ex ante private α -core strategies is $\{(AB, AA)\}$.

2. The set of ex ante coarse α -core strategies is $\{(AA, AA), (BB, AA)\}$.

In this case the set of the private α -core strategies does not contain the set of α -core core strategies independent of decision stages

Case 2. $c > a$

From the strategy bundle (AB, AB) , we obtain that the payoff of player 1 at state ω_1 is a , the payoff of player 1 at state ω_2 is c , and the expected payoff of player 2 is $(a + c)/2$. Any strategy bundles cannot improve upon (AB, AB) . Other strategy bundles are improved upon by (AB, AB) .

Remark 3

The set of non-measurable coarse α -core strategies is $\{(AB, AB)\}$.

Remark 4

The set of ex ante non-measurable coarse α -core strategies is $\{(AB, AB)\}$.

In this case we obtain that the private α -core strategy (AB, AA) does not belong to the set of non-measurable coarse α -core strategies independent of the measurability.

4 Concluding Remarks

This example suggests that the existence of private α -core does not imply the existence of coarse α -core independent of measurability or decision stages. For this reason, we pay attention to applying this existence result to coarse α -core.

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[抄 録]

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