

Assortment Function Model in the Marketing Structures

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1. Purpose

I, aiming at analyzing the structures of vertical markets, have constructed the models of the communication function necessary for demand and supply coordination. Starting with the link model by M. Hall, some premises were added for the model formulation. As a result, I got several new significant theoretical hypotheses⁽¹⁾.

I here concentrate on discussing assortment function, which is also necessary for demand and supply coordination, but is considered as basically an auxiliary function in this study. I analyze characteristics of structures in vertical market by formulation of the function model in this study.

2. Definition of the Assortment Function

As for the assortment function, its importance varies among researchers. Some think it is very crucial in vertical markets⁽²⁾ and some think it is simply an auxiliary and facilitating function⁽³⁾, or even an element of inventory and transportation functions⁽⁴⁾. For example, W. Alderson explained the whole structure

(1) Fumitaka Nishimura, *Ryutsu Kohzo Kaiseki Ron (An Analysis of Marketing Structures)*, Doyukan, 1999, pp.172-190 and pp.201-262.

(2) See M. Hall, "The Theory of Wholesale Distribution," in W. G. Moller and D. L. Wilemon (eds.), *Marketing Channel: A Systems Viewpoint*, Irwin, 1971, p.165 and pp.171-173.

of distribution channels with the social transfer of goods through a characteristic manipulation of the assortment concept⁽⁵⁾. Basically, this study treats it as a supplemental function in vertical market.

Regardless of the differences in the socioeconomic perception of the assortment function, the basic concept of assortment in this study shares with the most of the researchers' function list in the marketing.

Few of them, however, have succeeded to expand the assortment concept or principle and utilize it for explaining the mode or structure of vertical markets. This is mainly because the concept is so abstract and ambiguous that it is difficult to construct models for it. W. Alderson was exceptional as mentioned above. L. P. Bucklin⁽⁶⁾ also, by a different concept from Alderson's, linked the structure of vertical markets to the assortment function in a more concrete way and developed his assortment function model⁽⁷⁾.

Here, the assortment function model is constructed quite different from

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- (3) For a discussion of the assortment function as a facilitating or auxiliary function, see C. F. Phillips & D. J. Duncan, *Marketing: Principles and Methods*, Irwin, 1968, pp.32-34 and D. S. Warner, *Marketing and Distribution: An Overview*, McGraw-Hill, 1969, pp.55-57.
 - (4) L. P. Bucklin considers the sorting function to be in relation with the transit function and the inventory function. See L. P. Bucklin, *A Theory of Distribution Channel Structure*, Univ. of California (Special Publications), 1966, p.31.
 - (5) (i)W. Alderson, "Factors Governing the Development of Marketing Channels," in B. E. Mallen (ed.), *The Marketing Channel: A Conceptual Viewpoint*, Wiley, 1967, pp.38-39.
(ii)W. Alderson, "Factors Governing the Development of Marketing Channels," in W. G. Moller and D. L. Wilemon (eds.), *Marketing Channel: A Systems Viewpoint*, Irwin, 1971, pp.21-22.
(iii)W. Alderson, *Marketing Behavior and Executive Action: A Functionalist Approach to Marketing Theory*, Irwin, 1957, pp.201-211.
 - (6) For a model of the sorting function principle by L. P. Bucklin, see L. P. Bucklin, *op. cit.*, Chapter IV.
 - (7) B. R. Holdren is another researcher who succeeded in formulating an assortment model. See B. R. Holdren, *The Structure of a Retail Market and the Market Behavior of Retail Units*, Iowa State Univ. Press, 1968. Yukichi Arakawa introduces Holdren's model in his book: *Shogyo Kohzo to Ryutsu Gourika (Marketing Structures and Rationalization of Marketing)*, Chikura Shobo, 1969, pp.87-105.

Bucklin's, which is based on transportation, and more heavily on the stock function. I borrow the assortment concept from W. Alderson and partially the link principle. Also the concept of demand creation by P. A. Naert⁽⁸⁾, whose model was based on the concept of demand creation or availability by D. B. Montgomery and G. L. Urban⁽⁹⁾, is used as a part of the model in this study.

In socioeconomic vertical markets in the modern economy, mass-assorted products based on manufacturers convenience need to be adjusted for ultimate consumers' demand in size and assortment. This is obvious even without W. Alderson's indication⁽¹⁰⁾, because manufacturers sort commodities according to their production technology and ultimate consumers' demand is determined by the pattern of their consumption.

Therefore, W. Alderson called the difference of assortment demand between manufacturers and ultimate consumers the "discrepancy of assortment"⁽¹¹⁾. In this study, all kinds of activities which make up such basic discrepancies ("gaps of technological assortment") are collectively called "assortment function".

By M. Hall, for example, specific actions by firms to fill out gaps of assortment are recognized simply as the action of adjusting the size and the type of products⁽¹²⁾. W. Alderson, however, names the activities for filling the gap "sorting" and explains them as the following four steps of activity processes.

(1) Sorting Out: the activities which classify goods of the same quality by grading or sorting out goods of different qualities.

(2) Accumulation: the activities which form a large group of goods by accumulating a small amount of goods of the same quality.

(8) P. A. Naert, *Mathematical Models of Vertical Market Structures*, Unpublished Ph. D. Dissertation, Cornell Univ., 1970, pp.4-5, p.65 and pp.71-74.

(9) D. B. Montgomery & G. L. Urban, *Management Science in Marketing*, Prentice-Hall, 1969, pp.207-211 and pp.213-215.

(10) See (i)W. Alderson, *op. cit.*, p.38. (ii)W. Alderson, *op. cit.*, pp.19-21. (iii)W. Alderson, *op. cit.*, pp.215-216.

(11) (i)*Ibid.*, p.38. (ii)*Ibid.*, pp.19-21. (iii)*Ibid.*, pp.215-216.

(12) M. Hall, *op. cit.*, p.171 and p.173.

(3) Allocation: the activities which divide the large amount of goods of the same quality into small amounts, according to buyers' needs.

(4) Assorting: the activities which combine groups of goods of small amount of (3) into the one which consists of goods regarded as different quality but probably mutually relevant to one another⁽¹³⁾, according to the needs of ultimate consumers.

The above four procedures are necessary for smooth coordination of supply and demand in socioeconomic vertical markets. Which action is the most important depends on the characteristics of products and demand. Among the above four activities, "assorting" cannot be carried out by individual manufacturers producing a single or a few products, because to assort products from several firms is possible only by middlemen in vertical markets⁽¹⁴⁾. As such, M. J. Baker says, "sorting and assorting are functions carried out mainly by wholesalers and retailers, but also to some extent by manufacturers who perform some wholesale functions"⁽¹⁵⁾.

Therefore, we may be able to conclude that to assort products is the role of middlemen. In this study I assume this role of middlemen and I also assume that the resulting assorting satisfies ultimate consumers' needs. This saves ultimate consumers' time and labor in choosing certain assortments and facilitates higher availability of products. As a result, the assorting done by middlemen enhances ultimate consumers' purchase interest. In other words, it creates consumers' demand.

W. Alderson's four activities for filling the gap of assortment choice between manufacturers and ultimate consumers will also be the premises of this study. Accumulation and allocation are, however, two sides of the same activity.

(13) (i)W. Alderson, *op. cit.*, pp.38-39. (ii)W. Alderson, *op. cit.*, pp.21-22. (iii)W. Alderson, *op. cit.*, pp.201-211.

(14) See Fumitaka Nishimura, *op. cit.*, pp.165-166. Yukichi Arakawa, *op. cit.*, p.37. Kinnoyuke Yamanaka, *Maketingu Moderu (Marketing Model)*, Chikura Shobo, 1975, pp.35-36.

(15) M. J. Baker, *Marketing: Theory and Practice*, Macmillan, 1969, p.104.

Accumulation is a word for purchasers and allocation for the seller. But when purchasers decide to sell the accumulated products or when they become sellers, they will allocate them according to the needs of ultimate consumers.

After all, three activities: (1) sorting out, (2) accumulation and allocation (3) assorting make up the assortment function of this study. (1) sorting out and (2) accumulation and allocation are, needless to say, able to be operated by both manufacturers and middlemen, but (3) assorting, as mentioned above, is a role exclusive to middlemen.

By the way, the cost of performing any assortment functions is hypothetically calculated like the communication function model⁽¹⁶⁾ for the case where there is no middleman with the structure of production and consumption given. But as for (3) assorting cost, I will not apply the method which obtains the equilibrium number of middlemen by taking various structures of middlemen into account. The conditions of cooperation for (3) assorting are given differently from usual cases. Refer to Section 4 for this point.

The cost of the assortment function depends on the average cost of the above three activities, the number of manufacturers, of consumers, of the customers for segmented markets, of assortment activities carried out annually, of products distributed, and of classified groups, investment and so on.

3. Basic Model

3.1 Premises, Hypotheses and Symbols

In this section, I start to formulate the assortment function model consisting of the two models: the grading or sorting out model, and the accumulation or allocation model. Before that, I will set up premises or several hypotheses for the formulation as follows:

1) The basic constituent members of a vertical market, manufacturers, ultimate consumers and middlemen are labeled M, N and W_1 , respectively. Their numbers

⁽¹⁶⁾ See Fumitaka Nishimura, *op. cit.*, pp.172-189.

are m , n and w_{1i} , respectively.

2) The number of grades or classes in the said vertical market: G

3) Annual average consumption by product for each ultimate consumer: Q

4) Average number of customers for a segmented market: S_M

Market segmentation ratio: S_M/n

5) Average cost for each grade or classified category at the grading or sorting out activities: k_{A1}

6) k_{A1} varies by the number of goods transacted and is an increasing function of the number of products \bar{Q} .

7) The number of goods for each grade is assumed to be the same.

8) If the same profit is secured for manufacturers even when middlemen take over activities such as grading, sorting out, accumulating, and allocating goods instead of manufacturers, manufacturers do not forbid middlemen to intervene.

9) Manufacturers pay middlemen for the total socioeconomic cost necessary in the M-N structure in the form of a profit margin or a mark-up.

10) Each enterprise needs to invest for the activity of grading or sorting out and accumulation or allocation. Here, the amount of investment is F_A , and the profit ratio of necessary investment is r .

11) Annual number of goods transfer: K_T

12) The operational size of manufacturers and middlemen are taken into account into their averages.

3.2 Grading and Sorting Out Activities

As W. Alderson mentioned, grading and sorting out are activities which make goods of different quality or kind into small groups of goods of the same quality. This activity is very common for agricultural products.

Grading and sorting out in the M-N structure are conducted by manufacturers or producers themselves. Those in the M-W₁-N structure are carried out by middlemen, while they stock the goods supplied by manufacturers or producers.

Therefore as an additional premise we assume that middlemen are in charge of the storage function when the M-N structure shifts to the M-W₁-N structure.

Since the annual total transaction volume of goods in socioeconomic vertical markets is nQ , the annual average production quantity per each manufacturer or producer is

$$\frac{nQ}{m}. \quad (2-1)$$

In the M-N structure individual manufacturers and producers sort out the above quantity (2-1) into G grades from premise 2).

Since premise 7) assumes the number of goods in each grade is the same, the average number of goods per grade is

$$\overline{Q_{S0}} = \frac{nQ/m}{G} = \frac{nQ}{mG}. \quad (2-2)$$

Premise 5) gives the average cost per grading or selecting out as k_{A1} and the total number of grading by all manufacturers or producers is mG . Therefore the socioeconomic annual total cost TC_{AM} is

$$TC_{AM} = mG \times k_{A1} \times K_T, \quad (2-3)$$

where the average number of goods per grade ($\overline{Q_{S0}}$) in equation (2-3), implies equation (2-2).

What would happen if middlemen enter vertical markets and perform grading or sorting out for manufacturers or producers? In other words, the market structure is M-W₁-N. The annual average of products with which a middleman deal is

$$\frac{nQ}{w_{1i}}. \quad (2-4)$$

From premise 2) middlemen also perform grading or sorting out G times, and premise 7) assumes the number of goods in each grade is the same. Therefore the average number of goods per grade $\overline{Q_{S1}}$ is

$$\overline{Q_{S1}} = \frac{nQ/w_{1i}}{G} = \frac{nQ}{w_{1i}G}. \quad (2-5)$$

The socioeconomic annual total cost for grading in the M-W₁-N structure, TC_{AW} is derived from the average cost per grading in premise 5), k_{A1} and the total number of grading by all middlemen, $w_i G$.

$$TC_{AW} = w_i G \times k_{A1} \times K_T \quad (2-6)$$

where the average amount of goods per grade \overline{Q}_{S1} in equation (2-6), implies equation (2-5).

Here if k_{A1} in (2-3) and that in (2-6) are the same, cooperation conditions in premise 8) and 9) give the following:

$$m G k_{A1} K_T = w_i G k_{A1} K_T \quad (2-7)$$

$$\therefore w_i = m. \quad (2-8)$$

Equation (2-8) indicates that the equilibrium number of middlemen will reach the number of manufacturers.

In the real world, k_{A1} in equation (2-3) and that in equation (2-6) are probably different. Generally, it is rational that the average cost per grade depends on the number of grading by middlemen as mentioned in premise 6), and this cost function is increasing in the number of products.

Here let us assume k_{A1} as in Figure 1. This figure shows that the economy of scales applies to k_{A1} against the number of goods \overline{Q}_S . In other words k_{A1} increases as \overline{Q}_S increases, but its increasing rate diminishes.

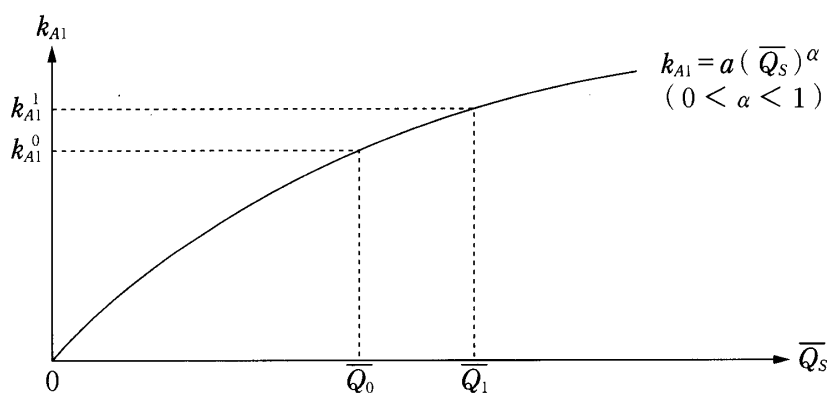


Figure 1 Relations between the cost per grade, k_{A1} and the average of goods per grade, \overline{Q}_S

The k_{A1} curve of Figure 1 is defined as follows⁽¹⁷⁾:

$$k_{A1} = a(\overline{Q_S})^\alpha, \quad 0 < \alpha < 1, \quad (2-9)$$

when k_{A1} on the M-N structure is k_{A1}^M and that in the M-W₁-N structure is k_{A1}^W ,

$$k_{A1}^M = a\left(\frac{nQ}{mG}\right)^\alpha, \quad 0 < \alpha < 1, \quad (2-10)$$

$$k_{A1}^W = a\left(\frac{nQ}{w_{1i}G}\right)^\alpha, \quad 0 < \alpha < 1, \quad (2-11)$$

where a is a parameter, α is an elasticity with the range, $0 < \alpha < 1$.

Again from the cooperation conditions of premise 8) and 9), the equilibrium number of middlemen w_{1i} is

$$w_{1i} = m \times \left(\frac{k_{A1}^M}{k_{A1}^W} \right). \quad (2-12)$$

3.3 Accumulation and Allocation Activities

Accumulation is to put together a small quantity of goods of the same quality graded or sorted out into larger groups as in 2.2. On the other hand, allocation is to divide the accumulated goods into smaller lots according to purchaser's demand.

Accumulation is, for example, to gather agricultural products to local markets for mass-distribution. In the case of manufactured goods, accumulation is carried out only within a company organization in the M-N structure. Manufacturers or producers, in principle, can not accumulate their goods outside their companies. On the other hand, division or allocation (commonly used terms among wholesalers and meaning "dividing into or unloading goods") can be performed both in the M-N structure and the M-W₁-N structure. In other words, in the M-N structure each manufacturer or producer breaks down goods into the quantity purchasers prefer and transport them in those quantities. In the M-W₁-N structure each middleman, who transported goods from manufacturers, does that operation.

Let us look at the above relations from a different direction. In the M-N

(17) For such cost functions, see Fumitaka Nishimura, *op. cit.*, pp.241-245.

structure, each manufacturer distributes its annual supply, nQ/m , K_T times in a smaller quantity which S_M number of customers demand. In the M- W_1 -N structure each manufacturer distributes its annual supply, nQ/m to w_{1i} middlemen, K_T times, and each middleman transports and accumulates those allocated goods, which is equal to nQ/w_{1i} per middleman, and also distributes nQ/w_{1i} , K_T times a year to n/w_{1i} number of their ultimate customers.

The above description again suggests that allocation and accumulation are different sides of the same operation. In other words, a purchaser accumulates goods, and a seller or a supplier divides goods. With the above discussion, let us derive the socioeconomic function cost for these activities in case of both the M-N structure and the M- W_1 -N structure.

In case of the M-N structure, the number of goods that a manufacturer distributes each time is

$$\frac{nQ/m}{K_T} = \frac{nQ}{mK_T}. \quad (3-1)$$

Since each manufacturer distributes the above number (equation (3-1)) to S_M number of its ultimate consumers, the quantity divided to each ultimate customer, by each manufacturer, $\overline{Q_{A0}}$, is

$$\overline{Q_{A0}} = \frac{nQ/mK_T}{S_M} = \frac{nQ}{mS_MK_T}. \quad (3-2)$$

Since each manufacturer allocates S_MK_T times, the number of allocation by all manufacturers is mS_MK_T . Therefore when the cost for allocation in the M-N structure, TC_{AM} , is

$$TC_{AM} = mS_MK_T \times k_{A2}, \quad (3-3)$$

where the average number of goods per allocation ($\overline{Q_{A0}}$) in equation (3-3), implies equation (3-2).

In the M- W_1 -N structure, a manufacturer needs first to allocate its goods, nQ/m , K_T times to the number of middlemen, w_{1i} . Therefore each manufacturer's average number of goods per allocation, $\overline{Q_{A1}^M}$, is

$$\overline{Q}_{A1}^M = \frac{nQ}{mw_{1i}K_T}. \quad (3-4)$$

Since the annual number of allocations by all manufacturers is $mw_{1i}K_T$ in the M-W₁-N structure, the total cost of allocation, TC_{AM} when the average cost per allocation is k_{A2} as that in the M-N structure is

$$TC_{AM} = mw_{1i}K_T \times k_{A2}, \quad (3-5)$$

where the average number of goods per allocation (\overline{Q}_{A1}^M) in equation (3-5), implies equation (3-4).

Each middleman transports and accumulates goods ($nQ/mw_{1i}K_T$) allocated by m number of manufacturers which is as in equation (3-4). The annual accumulation, nQ/w_{1i} , is distributed K_T times a year to n/w_{1i} number of customers.

Therefore each middleman's annual average of goods per allocation, \overline{Q}_{A1}^W is

$$\overline{Q}_{A1}^W = \frac{nQ/w_{1i}}{(n/w_{1i}) \times K_T} = \frac{Q}{K_T}. \quad (3-6)$$

Since the annual total number of allocations by all middlemen is nK_T , the total cost for middlemen, TC_{AW} , (when the average cost per allocation is also k_{A2}) is

$$TC_{AW} = nK_T \times k_{A2}, \quad (3-7)$$

where the average number of goods per allocation (\overline{Q}_{A1}^W) in equation (3-7), implies equation (3-6).

Based on the cooperation condition of premise 8) and 9), when manufacturers leave surplus for middlemen as the operation of allocation, the profit margin the former gives the latter will rationally be equation (3-3) minus (3-5). That is,

$$TC_{AM} = mS_M K_T k_{A2} - mw_{1i} K_T k_{A2}. \quad (3-8)$$

If k_{A2} is constant under both the M-N and the M-W₁-N structures, the equilibrium number of middlemen, w_{1i} , will be derived from equating equation (3-7) and equation (3-8). That is,

$$\begin{aligned} nK_T k_{A2} &= mS_M K_T k_{A2} - mw_{1i} K_T k_{A2}, \\ \therefore w_{1i} &= \frac{mS_M - n}{m} = S_M - \frac{n}{m}. \end{aligned} \quad (3-9)$$

Here, is the change in k_{A2} irrelevant to the quantity of goods per allocation? It will be rational to think that k_{A2} becomes higher as the number of goods per allocation increases, \overline{Q}_A . In other words we can apply the economy of scales to k_{A2} against \overline{Q}_A as we did grading in the previous section. Therefore let us assume equation (3-10).

$$k_{A2} = b(\overline{Q}_A)^\beta, \quad (3-10)$$

where b is a structure parameter and β is an elasticity greater than zero and less than one ($0 < \beta < 1$).

Then when k_{A2} in equation (3-3), (3-5), (3-7) are k_{A2}^M , k_{A2}^{MW} and k_{A2}^{WN} , respectively, each k_{A2} is as follows:

$$k_{A2}^M = b(nQ/mS_M K_T)^\beta, \quad (3-11)$$

$$k_{A2}^{MW} = b(nQ/mw_{1i}K_T)^\beta, \quad (3-12)$$

$$k_{A2}^{WN} = b(Q/K_T)^\beta. \quad (3-13)$$

Therefore when k_{A2} changes, the equilibrium number of middlemen, w_{1i} , is derived from the following manipulation⁽¹⁸⁾.

(18) In order to obtain the value of w_{1i} , we assume the following two relations:

$$n^{\beta-1} - m^{\beta-1}S_M^{\beta-1} = A,$$

$$n^{\beta-1}S_M^{\beta-1} = B.$$

Then we obtain

$$Aw_{1i}^{\beta-1} = B,$$

$$\therefore w_{1i}^{\beta-1} = \frac{B}{A}.$$

Taking logarithms,

$$\log w_{1i}^{\beta-1} = \log\left(\frac{B}{A}\right),$$

$$(\beta - 1)\log w_{1i} = \log B - \log A,$$

$$\log w_{1i} = \frac{\log B - \log A}{\beta - 1}.$$

Here, when

$$Y = \frac{\log B - \log A}{\beta - 1},$$

we obtain

$$w_{1i} = e^Y.$$

$$nK_T \left\{ b \left(\frac{Q}{K_T} \right)^\beta \right\} = mS_M K_T \left\{ b \left(\frac{nQ}{mS_M K_T} \right)^\beta \right\} - mw_{1i} K_T \left\{ b \left(\frac{nQ}{mw_{1i} K_T} \right)^\beta \right\}, \quad (3-14)$$

$$\therefore (n^{\beta-1} - m^{\beta-1} S_M^{\beta-1}) w_{1i}^{\beta-1} - n^{\beta-1} S_M^{\beta-1} = 0. \quad (3-15)$$

Investment is required for accumulation and allocation activities and grading activities. The amount of investment, F_A , is the same for all companies from premise 10). Suppose the payback period is l , the required rate of return on investment is r , the minimum required return on investment, A , is the same for every year and the scrap value of the equipment is ignored, then

$$A = \frac{F_A \times r}{1 - (1+r)^{-l}}. \quad (3-16)$$

As a result, TC_{AM} and TC_{AW} of the basic model, in which the division and accumulation models and the grading and sorting out models are combined, are as follows:

$$TC_{AM} = mGk_{A1}^M K_T + (mS_M K_T k_{A2}^M - mw_{1i} K_T k_{A2}^{MW}) + mA, \quad (3-17)$$

$$TC_{AW} = w_{1i} Gk_{A1}^W K_T + nK_T k_{A2}^{WN} + w_{1i} A. \quad (3-18)$$

The equilibrium number of middlemen for the basic model, w_{1i} , is obtained by equating equation (3-17) and equation (3-18).

4 . Assorting Cost and Excess Profit

As partially mentioned in Section 2, part of the cooperation conditions need to be revised for the assorting cost model in this section. This is because the assorting cost cannot be calculated for the M-N structure since manufacturers cannot assort goods from the other companies. Therefore additional premises for constructing the model of assorting cost are listed below.

13) Assorting goods from various companies stimulate or create ultimate consumer demand, i. e., increase the consumer's annual purchase amount of goods, $Q^{(19)}$.

(19) For discussion of demand creation, see D. B. Montgomery & G. L. Urban, *op. cit.*, pp.213-213.

14) The less manufacturers and ultimate consumers disperse geographically, the more demand is stimulated or created.

15) Therefore ultimate consumers' annual purchase amount of goods, Q , depends on the degree of geographical dispersion, D_L , and the number of combination assorting performed by middlemen, B .

16) Since assortment cannot possibly be conducted under the M-N structure, manufacturers indiscriminately allow middlemen to enter into the market as long as the same profit is guaranteed even when there is a structural change from the M-N structure to the M-W₁-N structure. When manufacturers get more profit in the M-W₁-N structure because of higher demand of ultimate consumers resulted from middlemen's assorting activity, they will give the gain to middlemen as an intermediation fee.

17) The average cost per combination (assorting) activities: k_{A3}

18) A manufacturer's average profit per additional product resulted from excess demand: h

Now let us go on to the construction of the assorting model. First we consider each manufacturer producing one kind of product in the M-N structure. Each manufacturer deals in this product with S_M number of customers. Therefore the number of product combination (assorting) in the M-N structure, B_M , is

$$B_M = 1. \quad (4-1)$$

In the M-W₁-N structure, each middleman deals with m number of manufacturers, and each would therefore have m kind of product combination. That is, the number of product combination (assorting) in the M-W₁-N structure, B_W , is

$$B_W = m. \quad (4-2)$$

The number of product combination in the latter structure is m times of that in the former.

By going back to premise 13), let us recall that the activity of assorting aims at creating and stimulating demand. Here I attempt to expand the demand creation

concept by P. A. Naert to assortment function. P. A. Naert thinks as follows⁽²⁰⁾:

The purchase amount of ultimate consumers, Q , depends on the number of middlemen, w_{li} , and the degree of geographical dispersion, D_L .

When there is only one middleman in the M-W₁-N structure, consumers would need to devote much of their energies acquiring the goods they like because they have no choice but purchasing from this middleman. In this sense, the availability of goods for consumers is low in such a market structure. If the number of middlemen increases, however, the availability will be better because of an increase in the number of purchasing points. Therefore, higher availability creates and stimulates demand. Therefore it is rational to consider that Q is an increasing function of w_{li} ⁽²¹⁾.

This study also incorporates the above P. A. Naert's demand creation concept with the following revisions and expansion. The more product combining middlemen perform, the less time and energies consumers use assorting for themselves. Furthermore, the variety of choice of goods stimulates consumers' purchase interest. Therefore demand per ultimate consumer, Q , is

$$Q = f(B). \quad (4-3)$$

P. A. Naert's argument continues as follows:

The amount of Q does not increase indefinitely with the increase of middlemen entering a market. It will approach a limit. In other words Q increases as w_{li} increases, but its increase rate diminishes and approaches a multiple of consumers' demand in the M-N structure, Q_M . This extremum, $(1 + \gamma)Q_M$, is regarded as the maximum of Q when w_{li} increases.

γ is positive when availability of goods does not become better because of the lack of middlemen entering a market, or when the increase in middlemen causes an increase in consumers demand. But when availability of goods becomes higher

(20) P. A. Naert, *op. cit.*, pp.72-73. Also see Fumitaka Nishimura, *op. cit.*, pp.124-126 for the discussion by Naert.

(21) P. A. Naert, *op. cit.*, p.72.

without middlemen's intermediation, or when direct transaction between manufacturers and consumers is very important, more goods will be transacted in the M-N structure than the M-W₁-N structure. This makes γ negative⁽²²⁾.

Then the question is how Q will increase as w_{1i} increases. Premise 14) assumes the increase in Q depend on the degree of geographical dispersion. When the constituent members of a vertical market are located in a small area, i. e. under the low degree of geographical dispersion, and $w_{1i} = 1$, an additional middleman does not increase goods availability very much. In this case, the single middleman would be able to handle the goods flow close to their maximum, $(1 + \gamma)Q_M$. But when the members of a vertical market disperse geographically, there is only one middleman for ultimate consumers and this inconvenient service of a middleman causes demand of goods to be far below the market's desired Q . Thus it implies that entrance of more middlemen is still necessary so that the demand of goods is going to $(1 + \gamma)Q_M$ ⁽²³⁾.

As a result of the above argument, P. A. Naert expressed the relations of Q with the two variables as follows⁽²⁴⁾:

$$Q = (1 + \gamma)Q_M \frac{ew_{1i}}{1 + ew_{1i}}, \quad (4-4)$$

where $e > 0$ and $w_{1i} > 0$.

e is an indicator of how the constituent members of a vertical market disperse geographically. When the market dispersion is very small, e is large and gives small changes in the value of $ew_{1i} / (1 + ew_{1i})$. In this case, when w_{1i} is small, Q is already close to $(1 + \gamma)Q_M$. On the other hand, e is very small under a highly geographically dispersed vertical market, and therefore w_{1i} needs to be larger to make Q close to $(1 + \gamma)Q_M$. Figure 2 shows the relation between Q and w_{1i} indicated in equation (4-4) for three values of e .

(22) *Ibid.*, p.72.

(23) *Ibid.*, p.72.

(24) *Ibid.*, p.73.

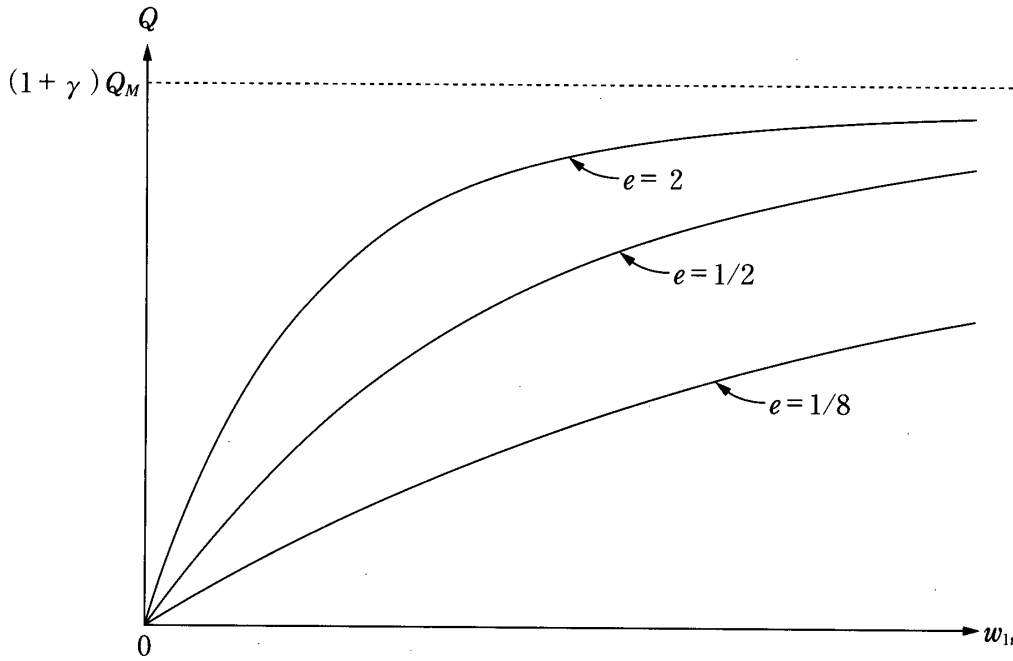


Figure 2 P. A. Naert, *Mathematical Models of Vertical Market Structures*, Unpublished Ph. D. Dissertation, Cornell Univ., 1970, p.73.

In the model of this study, Q is also considered to change according to the degree of e (geographical dispersion). But P. A. Naert showed only a notion that its degree is low when e is large and vice versa. He did not define “geographical dispersion”, e concretely. Therefore let us define his geographical dispersion parameter, e , by D_L as follows:

$$e = f\left(\frac{1}{D_L}\right), \quad (4-5)$$

$$\therefore e = c \times \frac{1}{D_L}, \quad (4-6)$$

where C is a parameter, needless to say, the geographical dispersion, D_L , of this study indicates the changes in distance between constituent members of a vertical market resulted from the entry of the middleman levels, not a two-dimensional dispersion among middlemen.

Therefore Q does not depend on the number of middlemen directly. As a result of the above discussion, the annual average demand per ultimate consumer, Q , is defined according to the number of product combination, B , and the degree of

geographical dispersion D_L , as follows:

$$Q = (1 + \gamma) Q_M \times \frac{dB \times c/D_L}{1 + dB \times c/D_L}, \quad (4-7)$$

$$= (1 + \gamma) Q_M \times \frac{c \times dB}{D_L + c \times dB},$$

Letting $g = cd$,

$$Q = (1 + \gamma) Q_M \times \frac{gB}{D_L + gB}, \quad (4-8)$$

where g is a parameter, $D_L > 0$, and $B \geq 1$.

When $B = 0$ in the above equation (4-8), that is, when there is no product combination, the right-hand side of the equation is zero. This indicates the situation in which there is no product to be distributed in either the M-N and the M-W₁-N structure.

When $B = 1$, that is, when there is only a kind of product combination, D_L is the only determinant for Q . This supports the argument for equation (4-1). That is, in the M-N structure when $B = 1$, Q mostly depends on D_L . Thus equation (4-8) explains the characteristics of Q rather well.

When Q is Q_{MN} in the M-N structure and Q_{WN} in the M-W₁-N structure, each Q is written as follows:

$$Q_{MN} = (1 + \gamma) Q_M \times \frac{g}{D_0 + g}, \quad (4-9)$$

$$Q_{WN} = (1 + \gamma) Q_M \times \frac{gm}{(D_0/2) + gm}, \quad (4-10)$$

where D_0 indicates the degree of geographical dispersion in the M-N structure and is defined as follows:

$$D_0 = \frac{\sum_{i=1}^m \left\{ \sum_{j=1}^{S_M} \frac{|M_i - N_j|}{S_M} \right\}}{m}. \quad (4-11)$$

The relation between D_L and the number of the middleman levels, L , is

$$D_L = \frac{D_0}{L+1}. \quad (4-12)$$

When $\gamma \geq 0$, since $m \geq 1$, $D_0 \geq D_1$, and $g \geq 0$,

$$Q_{MN} < Q_{WN}.$$

Through the transition from the M-N structure to the M-W₁-N structure, middlemen would strengthen the assorting activities and lower geographical dispersion, which would eventually increase demand, The increase in demand, Q_{MW} , is as follows:

$$\begin{aligned} Q_{MW} &= Q_{WN} - Q_{MN}, \\ &= (1 + \gamma) Q_M \left\{ \frac{gm}{(D_0/2) + gm} - \frac{g}{D_0 + g} \right\}, \end{aligned} \quad (4-13)$$

$$= (1 + \gamma) Q_M \left\{ \frac{gD_0(2m-1)}{(D_0 + 2gm) + (D_0 + g)} \right\}. \quad (4-14)$$

If the demand of ultimate consumers increased by the amount of equation (4-14) from the original demand in the M-N structure as a result of product combinations by middlemen, manufacturers would naturally gain excess profit, Π_E , which means excess surplus. When the average profit from a product for a unit of excess demand is h , Π_E is

$$\Pi_E = Q_{MW} \times h \times n. \quad (4-15)$$

On the other hand, we also need to consider the cost of assorting conducted by middlemen. Middlemen collect m number of goods from manufacturers to have m combinations. Among m combinations, they assort some combinations according to their customers taste and deliver them to the customers.

In this case, how do we define the number of product combination by middlemen? Suppose that the number of product combinations for consumers by middlemen is T and, without loss of generality and for simplicity, that that number is equivalent to the average number of product combinations per ultimate consumer. These assumptions make possible for us to apply the overlap concept⁽²⁵⁾

⁽²⁵⁾ See Fumitaka Nishimura, *op. cit.*, pp.202-212.

of the communication function to the model of this paper. But since information overlap, $T^{(26)}$, in communication function model includes double-counted information exchange for response, T in assorting activities will become as follows⁽²⁷⁾:

$$T = \frac{mS_M + n}{n} - 1 = \frac{mS_M}{n} \quad (4-16)$$

This equation (4-16) implies the average number of product combinations per ultimate consumer by each middleman because of the above assumption. The number of ultimate consumers per middleman is n/w_{1i} and therefore the average possible number of product combinations per middleman is

$$T \times n/w_{1i} = \left(\frac{mS_M}{n} \right) \left(\frac{n}{w_{1i}} \right) \quad (4-17)$$

Then the annual average number of socioeconomic product combinations for all middlemen, T_W , is

$$T_W = mS_M K_T \quad (4-18)$$

The average cost for product combination is k_{A3} based on premise 17), the total socioeconomic cost for middlemen assorting, TC_{AW} , is

$$TC_{AW} = k_{A3} mS_M K_T \quad (4-19)$$

From premise 16), the above cost (equation (4-19)) needs to be compensated by the excess profits, Π_E . Therefore,

$$\Pi_E = TC_{AW} \quad (4-20)$$

5 . The Comprehensive Model of the Assortment Function

The equations below are the comprehensive models including (1) the activity of grading or sorting out, (2) the activity of allocation or accumulation, and (3) the activity of assorting.

$$TC_{AM} = mK_T \{ Gk_{A1}^M + S_M k_{A2}^M - w_{1i} k_{A2}^{MW} \} + (Q_{WM} - Q_{MN}) nh + mA, \quad (5-1)$$

$$TC_{AW} = w_{1i} (Gk_{A1}^W K_T + A) + K_T (nk_{A2}^{WN} + mS_M k_{A3}). \quad (5-2)$$

(26) For information overlap T (equation (6-26)), see *ibid.*, p.210.

(27) *Ibid.*, pp.201-212.

By equating the above equations (5-1) and (5-2), we obtain the equilibrium number of middlemen, w_{1i} .

$$\therefore w_{1i} = \frac{\{mGk_{A1}^M K_T + mK_T S_M k_{A2}^M + (Q_{WN} - Q_{MN})nh + ma\} - K_T(nk_{A3}^{WN} + mS_M k_{A3}^M)}{(Gk_{A1}^W K_T + A + mK_T k_{A3}^{MW})}, \quad (5-3)$$

where

$$k_{A1}^M = a(nQ_{MN}/mG)^\alpha,$$

$$k_{A1}^W = a(nQ_{WN}/w_{1i}G)^\alpha,$$

$$k_{A2}^M = b(nQ_{MN}/mS_M K_T)^\beta,$$

$$k_{A2}^{MW} = b(nQ_{WN}/mw_{1i}K_T)^\beta,$$

$$k_{A2}^{WM} = b(Q_{WN}/K_T)^\beta,$$

$$A = \frac{F_A \times r}{I - (1 + \gamma)^{-l}},$$

$$Q_{MN} = g(1 + \gamma)Q_M/(D_0 + g),$$

$$Q_{WN} = gm(1 + \gamma)Q_M/((D_0/2) + gm),$$

$$D_0 = \frac{\sum_{i=1}^m \left\{ \sum_{j=1}^{S_M} \frac{|M_i - N_j|}{S_M} \right\}}{m}$$

6 . Conclusion: the Summary of Models

We show the important premises for the assortment function model and its implications in this section. Premises 1 to 16 below are those discussed in the course of the model construction. Implications 17 to 21 below are theoretical hypotheses derived from the models constructed.

1 . The assortment function consists of three activities: grading or sorting out, allocation or accumulation, and assorting (See Section 2).

2 . Among the above activities, only the assorting activity is considered behavior characteristic to middlemen (See Section 2).

3 . The assorting activity stimulates and creates the demand of the ultimate consumers (See Section 2).

4 . The average cost of each grading activity is an increasing function of the number of goods with a diminishing increasing rate (See Subsection 3.1).

5 . Each firm needs investment in order to carry out the assortment function (See Subsection 3.1).

6 . In order that middlemen perform the assortment function in the $M-W_1-N$ structure, they also must have the stock function (See Subsection 3.2).

7 . In the $M-N$ structure, the number of grading is a function of the number of manufacturers, grades, and the annual number of transportation of goods (See Subsection 3.2).

8 . In the $M-W_1-N$ structure, the number of grading is a function of the number of middlemen, grades, and the annual number of transportation of goods (See Subsection 3.2).

9 . The average cost of each allocation activity is an increasing function of the transacted number of goods with a diminishing increasing rate (See Subsection 3.3).

10 . The number of allocation activities in the $M-N$ structure is a function of the number of manufacturers and of the number of customers in a segmented market and the annual number of shipments (See Subsection 3.3).

11 . In the $M-W_1-N$ structure, the number of allocation activities in the part of $M-W_1$ is a function of the number of manufacturers and middlemen and the annual number of shipments by manufacturers, while the number of allocations activity in the part of W_1-N is a function of the number of ultimate consumers and shipments by middlemen (See Subsection 3.3).

12 . The annual purchase amount of ultimate consumers is regarded as a function of the degree of geographical dispersion between sellers and consumers and the number of product combinations (See Section 4).

13 . The lower the degree of geographical dispersion between sellers and consumers, the higher the annual purchase of ultimate consumers (See Section 4).

14 . The annual purchase volume of ultimate consumers increases as the

number of product combinations increases, and converging at a certain maximum level (See Section 4).

15. From premises 12, 13, and 14, we can say that the annual purchase of ultimate consumers increases when the degree of geographical dispersion is low and there are many product combinations, because these stimulate ultimate consumers purchase interest (See Section 4).

16. The cost of assorting, in principle, is explained by the excess profit of manufacturers (See Section 4).

17. In case middlemen carry out accumulation and allocation activities in place of manufacturers, the equilibrium number of middlemen is explained by the number of manufacturers, ultimate consumers, and the customers in a segmented market when investment costs are ignored. This is so when the average cost is the same for the above activities in the M-N structures and M-W₁-N structures (See Subsection 3.3).

18. The equilibrium number of middlemen heavily depends on the number of customers in a segmented market. The greater the latter, the greater the former (See Subsection 3.3).

19. The necessary condition that a positive number of middlemen exist is provided that both the number of manufacturers and ultimate consumers are two or more than two (See Subsection 3.3).

20. The equilibrium number of middlemen increases as the number of manufacturers increases or that of ultimate consumers decreases (See Subsection 3.3).

21. The equilibrium number of middlemen increases as the number of the customers in a segmented market approach the number of ultimate consumers (See Subsection 3.3).